

# On E0 Transitions in even-even nuclei

Vladimir P. *Garistov*<sup>1</sup>, A.A. *Solnyshkin*<sup>2</sup>, I. *Adam*<sup>2</sup>,  
O.K. *Egorov*<sup>3</sup>, A. *Islamov*<sup>3</sup>, V.I. *Silaev*<sup>3</sup>, D.D. *Bogachenko*<sup>3</sup>

Institute for Nuclear Research and Nuclear Energy, Sofia, *Bulgaria*<sup>1</sup>

Joint Institute for Nuclear Research, Dubna, *Russia*<sup>2</sup>

Institute for Theoretical and Experimental Physics, Moscow, *Russia*<sup>3</sup>

## Abstract

The reanimation of the investigations dedicated to  $0^+$  states energies and  $E0$  transitions between them is provoked by new and more precise experimental technics that not only made revision of the previous data but also gave a possibility to obtain a great amount of new  $0^+$  states energies and conversion electrons data. We suggest one phenomenological model for estimation of the  $E0$  transition nuclear matrix elements. Recently theoretical calculations[1] predicted existence of a  $0^+$  state with energy 0.68 MeV in  $^{160}\text{Dy}$  nucleus. Powerful enough arguments in favor of existence of 681.3 keV state in  $^{160}\text{Dy}$  nucleus are presented.

## 1 Introduction

Nature of low lying  $0^+$  states bands in deformed nuclei remains a mystery under debate. The improvements in technology have remedied the situation by enabling spectroscopy, reactions, and life-time measurements of a large number of  $K^\pi=0^+$  bands that were previously inaccessible in nuclei [2]. Some authors point out importance to study anharmonic effects in microscopic way in deformed nuclei [3], quadrupole and pairing vibrational modes in conversional electrons and internal pair decay [4], or exact diagonalization in the restricted space of collective phonons of different types [6].

The energies and electromagnetic decay properties of the excited  $0^+$  states are important in determining the applicability and test of the models - Shell model, Cluster-vibrational model, Quasi-particle - phonon model, a deformed configuration mixing shell model, Interacting boson approximation, pairing quadrupole correlations,  $O(6)$  limit of  $IBA$ .

There are some but poor enough calculations [4], [7], [11], [12] devoted to estimation of  $E0$  nuclear matrix elements  $\rho^2$  between different  $0^+$  states in the same nucleus. For instance in [12] we find that  $\rho^2(0_2^+ \rightarrow 0_1^+)$  is very small in compare with  $\rho^2(0_3^+ \rightarrow 0_1^+)$  that indicates the more collective nature of the  $0_2^+$  state. It should be very important to determine the half-lives of the  $0^+$  states

that would allow more definite conclusions on the structures of the excited  $0^+$  states [11]. Often the first excited  $0^+$  state in nuclei is considered as less collective than the next by increasing energy states. In some even-even nuclei the first by energy excited state is not necessarily the lowest state by degree of collectivity [12]. For instance observed in  $^{158}\text{Gd}$   $0^+$  state with the excitation energy 0,2548 MeV ( $n = 20$ ) [1] is also much more collective than the  $0^+$  state with energy 0,5811 ( $n = 1$ ) MeV.

## 2 E0 transition matrix elements

The separation of the E0 conversion probability into electronic and nuclear factors is not as well defined as for the conversion of higher multipoles, nor is the electronic factor,  $\Omega$ , completely independent of nuclear properties. Physically, the monopole transition interaction vanishes except while the electron is within the nuclear charge distribution, and hence it is the electron wave functions within the nucleus which enter into the calculation of  $\Omega$ . These in turn, depend on the average static nuclear charge distribution. In this paper we will estimate the transition probability without mutual influence of the electronic and nuclear wave functions (that really may occur very important). Nevertheless using this approximation we can feel the gross-behavior of the transition probabilities coming from the transition charge density distribution. Let us consider the simple description of the K-electrons conversion process starting with the Hamiltonian [17]

$$H = H_{nuc} + H_{elect} - \sum_{p,e} \frac{\alpha}{|r_p - r_e|} \quad (1)$$

$$H' = - \sum_{p,e} \frac{\alpha}{|r_p - r_e|} \quad (2)$$

$$\begin{aligned} \langle i | H'(L=0) | f \rangle = \\ - \sum_{p,e} \alpha \left[ \int d\tau_{nuc} \int_0^{r_p} d\tau_e \phi_f^* \psi_f^* \frac{1}{r_p} \phi_i \psi_i + \int d\tau_{nuc} \int_{r_p}^{\infty} d\tau_e \phi_f^* \psi_f^* \frac{1}{r_e} \phi_i \psi_i \right] \end{aligned} \quad (3)$$

Using the approximation [17]

$$\sum_p \int d\tau' \phi_j^* \phi_j \delta(r'_p - r) \quad (4)$$

and electron initial wave function  $\sim e^{-ar}$  and  $\sim e^{i\mathbf{k}\mathbf{r}}$  in infinity. In the case of cut-off charge density distribution  $d_0 \Theta(R-r)$  for K-electrons we find the result of the above integration (3) as

$$F_{nuc,el}(k, R) = \frac{16\pi^2 \alpha (kR (k^2 R^2 + 3) \cos(kR) - 3 \sin(kR))}{3k^5} \quad (5)$$

It is very important for our further consideration that this nucleus-electron factor depends on the nucleus size  $R$  and electron impulse defined by nuclear transition energy. To make the sense of this sentence more understandable let  $d(r, R)$  be any nuclear characteristic that depends on nuclear size  $R$ . Thus we can write

$$d(r, R) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d(r, R_0 + x) e^{ip(x - \Delta R(b^+, b))} dp dx \quad (6)$$

and following the receipt of paper [18] we introduce the monopole degrees of freedom and then the expectation values of this characteristic between different collective states is determined by matrix elements

$$\langle n_1 | d(r, R) | n_2 \rangle \quad (7)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d(r, R_0 + x) e^{ipx} \langle n_1 | e^{-ip\Delta R(b^+, b)} | n_2 \rangle dp dx \quad (8)$$

For illustration of this approach let take  $d(r, R) = d_0 \theta(R - r)$  and calculate the matrix elements for the ground state remembering that

$$\langle 0 | e^{-ip\Delta R(b^+, b)} | 0 \rangle = e^{\frac{-p^2 S^2}{2}}$$

Performing integration by  $dp$  and  $dx$  in (7) we have

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d_0 \theta(R - r - x)}{2\pi} e^{ipx - \frac{p^2 S^2}{2}} dp dx = \frac{1}{2} d_0 \text{Erfc}\left(\frac{r - R}{\sqrt{2}S}\right) \quad (9)$$

And as a result of integration by  $dp dx$  we obtain instead of starting cut-off function the function with surface diffuseness (see Figure ??):

$$\theta(R - r) \Rightarrow \frac{1}{2} d_0 \text{Erfc}\left[\frac{r - R_0}{\sqrt{2}S}\right] \quad (10)$$

And the corresponding mean square radius is :

$$\langle r_{ms}^2 \rangle_{00}^{1/2} = \sqrt{\frac{0.6R_0^5 + 6R_0^3 S^2 + rR_0 S^4}{R_0^3 + 3R_0 S^2}} \quad (11)$$

It is clear that in the case of  $S = 0$  we have the cut-off density mean square radius

$$\langle r_{ms}^2 \rangle_{00}^{1/2} = \sqrt{\frac{3}{5}} R_0 \quad (12)$$

Small vibrations of nuclear shapes around equilibrium can give rise to physical states at low to moderate excitation energies.

The description of the vast amount of experimental data on the low-lying collective spectra of even-even nuclei in the rare-earth and actinide regions is still a problem of particular interest in the nuclear structure physics. The classifications of this data is mainly done from a "horizontal perspective" in sequences

of nuclei where the investigated nuclear characteristics are empirically studied as functions of the numbers of their valence nucleons. The experimental information on a large sequences of states with  $J^\pi = 0^+$ , is in some cases enough to use a kind of a statistical approach for the distribution. Many well studied nuclei can be listed around the nuclear chart. The theoretical approaches that are able to explain and correctly describe all the data in the same nucleus in this respect are seemingly in debt to the experiment. Let us try to less our debt at least with the  $0^+$  bans starting with simple Hamiltonian

$$H = \alpha R_+ R_- + \beta R_0 R_0 + \frac{\beta \Omega}{2} R_0 \quad (13)$$

These operators was constructed with the pairs of fermion operators  $a^\dagger$  and  $a$  of the fermions placed at sub-shell  $j$ .

$$\begin{aligned} R_+ &= \frac{1}{2} \sum_m (-1)^{j-m} \alpha_{jm}^\dagger \alpha_{j-m}^\dagger, \\ R_- &= \frac{1}{2} \sum_m (-1)^{j-m} \alpha_{j-m} \alpha_{jm}, \\ R_0 &= \frac{1}{4} \sum_m (\alpha_{jm}^\dagger \alpha_{jm} - \alpha_{j-m} \alpha_{j-m}^\dagger), \\ [R_0, R_\pm] &= \pm R_\pm, \quad [R_+, R_-] = 2R_0 \end{aligned} \quad (14)$$

Further applying the Holstein-Primakoff transformation to the operators  $R_+$ ,  $R_-$  and  $R_0$

$$R_- = \sqrt{2\Omega - b^\dagger b} b \quad R_+ = b^\dagger \sqrt{2\Omega - b^\dagger b} \quad R_0 = b^\dagger b - \Omega \quad (15)$$

$$[b, b^\dagger] = 1 \quad [b, b] = [b^\dagger, b^\dagger] = 0 \quad b|0\rangle = 0 \quad |n\rangle = \frac{1}{\sqrt{n!}} b^n |0\rangle \quad (16)$$

the initial Hamiltonian (13) written in terms of ideal bosons has the form:

$$H = A b^\dagger b - B b^\dagger b b^\dagger b. \quad (17)$$

Thus the energy spectrum produced by Hamiltonian (17) is the parabolic function of the number of monopole ideal bosons  $n$

$$E_n = A n - B n^2 + C \quad (18)$$

Such a classification of large amount of experimental data in terms of integer classification parameter recently has been done based on phenomenological monopole part of collective Hamiltonian for single level approach written in terms of boson creation and annihilation operators  $R_+$ ,  $R_-$  and  $R_0$

Now we can label every  $K^\pi = 0^+$  state by an additional characteristic  $n$  - number of monopole bosons determining it's collective structure. The parameters  $A$  and  $B$  of (17) are evaluated by fitting the experimental energies of the different  $0^+$  states of a given nucleus to the theoretical ones applying all possible permutations of the classification numbers  $n$  and extracting the distribution corresponding to the minimal value of  $\chi$ -square [5]. The analysis based on phenomenological collective Hamiltonian (17) have shown that the experimental

energies of low lying excited  $0^+$  states in nuclei can be rearranged in a manner in which the energies of these states are distributed by number of collective excitations with parabolic distribution function  $E_n = An - Bn^2 + C$ .

Estimating further the E0-transition nuclear matrix elements between different excited  $0^+$  states  $\frac{1}{\sqrt{m!}}(b^\dagger)^m|0\rangle$  and  $\frac{1}{\sqrt{n!}}(b^\dagger)^n|0\rangle$  in the same nucleus we automatically have the transition energy values  $E(n) - E(m)$ . Now we know how to proceed - first we must calculate the transitional matrix elements.

$$f(m, n, p, w) \rightarrow \langle n | e^{-ip\Delta R(b^\dagger, b)} | m \rangle \quad (19)$$

$$f(m, n, p, w) = \langle n | e^{-ip\Delta R(b^\dagger, b)} | m \rangle \frac{1}{\sqrt{n!m!}} \langle 0 | b^n e^{-ip\Delta R(b^\dagger, b)} (b^\dagger)^m | 0 \rangle \quad (20)$$

Using obtained in [19] expressions

$$\sum_{l=0}^{n-1} \frac{m!}{(m-n+l)!} \binom{n}{l} (b^\dagger)^{m-n+l} b^l \quad n \leq m \quad (21)$$

$$\sum_{l=0}^{m-1} \frac{n!}{(n-m+l)!} \binom{m}{l} (b^\dagger)^l b^{n-m+l} \quad n \geq m \quad (22)$$

we can find from (20)

$$f(m, n, p, w) = \frac{e^{\frac{p^2 R_0^2 w^2}{2}}}{\sqrt{n!m!}} \sum_{k=0}^{\infty} (R_0 w)^{2k+m-n} (ip)^{2k+m-n} \frac{(m+k)!}{k!(m+k-n)!} \quad (23)$$

and summation by k gives:

$$f(m, n, p, w) = \frac{e^{\frac{p^2 R_0^2 w^2}{2}} (ip)^{m-n} (R_0 w)^{m-n} \Gamma(1+m) {}_1F_1(1+m, 1+m-n, -p^2 R_0^2 w^2)}{2\pi \sqrt{m!n!} \Gamma(1+m-n)} \quad (24)$$

Finally the nuclear E0 transitional matrix elements is nothing but:

$$\rho_{mn} = \frac{A_{norm}}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_{nuc,el}(k, R_0 + x) e^{ipx} f(m, n, p, w) dp dx \quad (25)$$

### 3 Results and Conclusion

$$f(m, m, p, w) = \frac{e^{\frac{p^2 w^2}{2}} \Gamma(m+1) {}_1F_1(m+1; 1; -p^2 w^2)}{2\pi} \quad (26)$$

$$f(m, m-1, p, w) = \frac{ie^{\frac{p^2 w^2}{2}} p w \Gamma(m+1) {}_1F_1(m+1; 2; -p^2 w^2)}{2\pi} \quad (27)$$

$$f(m, m-2, p, w) = -\frac{e^{\frac{p^2 w^2}{2}} p^2 w^2 \Gamma(m+1) {}_1F_1(m+1; 3; -p^2 w^2)}{4\pi} \quad (28)$$

$$f(m, 0, p, w) = \frac{e^{-\frac{1}{2}p^2w^2}(ip)^m w^m}{2\pi} \quad (29)$$

For chosen  $m$  and  $n$  we can perform integration by  $dpdx$  in (25). For instance

$$\begin{aligned} \rho_{1 \rightarrow 0} &= \frac{5}{3}k^4\pi w^9 - \frac{20}{3}k^4\pi R_0^2w^7 + \frac{16}{3}k^2\pi w^7 - \frac{10}{3}k^4\pi R_0^4w^5 \\ &\quad + 16k^2\pi R_0^2w^5 - 8\pi w^5 - \frac{4}{9}k^4\pi R_0^6w^3 + \frac{16}{3}k^2\pi R_0^4w^3 \\ &\quad - 16\pi R_0^2w^3 - \frac{1}{63}k^4\pi R_0^8w + \frac{16}{45}k^2\pi R_0^6w - \frac{8}{3}\pi R_0^4w \\ \rho_{2 \rightarrow 1} &= -\frac{50}{3}k^4\pi w^9 - \frac{160}{3}k^4\pi R_0^2w^7 + \frac{128}{3}k^2\pi w^7 - 20k^4\pi R_0^4w^5 \\ &\quad + 96k^2\pi R_0^2w^5 - 48\pi w^5 - \frac{16}{9}k^4\pi R_0^6w^3 + \frac{64}{3}k^2\pi R_0^4w^3 \\ &\quad - 64\pi R_0^2w^3 - \frac{2}{63}k^4\pi R_0^8w + \frac{32}{45}k^2\pi R_0^6w - \frac{16}{3}\pi R_0^4w \\ \rho_{4 \rightarrow 1} &= -\frac{400}{3}k^4\pi w^9 - 320k^4\pi R_0^2w^7 + 256k^2\pi w^7 - 80k^4\pi R_0^4w^5 \\ &\quad + 384k^2\pi R_0^2w^5 - 192\pi w^5 - \frac{32}{9}k^4\pi R_0^6w^3 + \frac{128}{3}k^2\pi R_0^4w^3 - 128\pi R_0^2w^3 \end{aligned}$$

The results for different  $m$  and  $n$  are analytical but because of large algebra we wont present all of them here.

For finding-out of existence of mentioned above  $0^+$  state with energy 0.68 MeV in  $^{160}\text{Dy}$  nucleus we measure  $\beta$ -spectrograms of [20] DLNP JINR for fractions Er (two photographic plates) and Ho (one photographic plate) using universal installation MAC-1 in ITEP [21]. At the analysis it was found out, that in all three photographic plates to the left of known line EIK with energy 682.3 keV below by energy on 1 keV, the peak comparable by intensity with the specified line is confidently observed. Our attempts to carry the mentioned peak to a conversion line or to any of known from the literature [22] transitions in  $^{160}\text{Dy}$  nucleus have not crowned with success. Then we proposed, that this peak is probably caused by new transition with energy 681.3 keV, unloading the corresponding new raised state with energy 681.3 keV to the ground state. Except for the specified state, from experiment the states with excitation energies 1280.0, 1456.7, 1708.2 and 1952.3 keV are known. Considering, that from these levels transitions to the entered by us 681.3 keV level are possible, we have undertaken searches of such transitions. As a result such transitions with energy 1822.5 1822.4(3)  $I = 0.24$ , between  $2^+$  state 2503.8 keV and  $0^+$  state 681.3 keV, and the transition from 681.3 keV state to  $2^+$  with energy 86.8 keV (594.5 and  $I < 0.3$ ) have been found out. In spite of that this facts already are powerful enough argument in favor of existence of a state 681.3 keV in a

nucleus  $^{160}\text{Dy}$ , we proceed the searches of other transitions.

### Caption to Figures

**Figure 1.** Density distributions before and after involving the collective degrees of freedom.

**Figure 2.** The behavior of calculated matrix elements  $\rho_{m \rightarrow 0}^2$  ;  $m$  is the number of monopole bosons constructed corresponding  $0^+$  excited state.

**Figure 3.** The behavior of calculated matrix elements  $\rho_{m \rightarrow m-1}^2$  ;  $m$  is the number of monopole bosons constructed corresponding  $0^+$  excited state

**Figure 4.** The behavior of calculated matrix elements  $\rho_{m \rightarrow m-2}^2$  ;  $m$  is the numbers of monopole bosons constructed corresponding  $0^+$  excited state.

**Figure 5.** The behavior of calculated matrix elements  $\rho_{m \rightarrow m-3}^2$  ;  $m$  is the number of monopole bosons constructed corresponding  $0^+$  excited state.

**Figure 6.**  $0^+$  states of  $^{160}\text{Dy}$  distributed in parabola. Red line presents our calculations, red circles point the region of predicted states and the stars are experimental data.

**Figure 7.**  $0^+$  states of  $^{158}\text{Gd}$  distributed in parabola. Red stars present our calculations and blue squares - experiment.

## References

- [1] V. Garistov, Rearrangement of the Experimental Data of Low Lying Collective Excited States, Proceedings of the XXII International Workshop on Nuclear Theory, ed. V. Nikolaev, Heron Press Science Series, Sofia, (2003), 305;
- [2] S.R. Leshner, A. Aprahamian et al. Phys. Rev. C66 051305 (R); A. Aprahamian, Phys. Rev. C 65, 031301(R), (2002).
- [3] B. Silvestre-Brac and R. Piepenbring, Phys. Rev. C 16, N4 (1977) 1638 ; Phys. Rev. C 17, N1 (1978) 1978 .
- [4] A. Passoja, J. Kantele, M. Luontama, J. Kumpulainen, R. Julin, P. O. Lipas and P.Toivonen, Physics Letters 124B, N3,4 (1983) 157
- [5] Vladimir P. Garistov e-Print Archive: nucl-th/0309058 ,Sep 2003; V. P. Garistov, Nuclear Theory, Proceedings of the XXII Workshop on Nuclear Theory, edited by V. Nikolaev, Heron Press Science Series, 305-311 V.P.Garistov IJMP-E 4 N2 1995 , 371; V.P.Garistov, A.Georgieva , H. Ganev,Algebraic Methods in Nuclear Theory, edited by Anton A. Antonov, Heron Press , Sofia, 2002; V.P.Garistov JMP-E4 N2 1995 , 371
- [6] J. Kantele, M. Luontama,W. Trzaska, R. Julin, A. Passoja, Physics Letters 171B, N2,3 (1986) 151
- [7] A.Passoja, R. Julin, J. Kantele, J. Kumpulainen, M. Luontama, and W. Trzaska, Nucl. Phys. A438 (1985) 413

- [8] K. Heyde, R.A. Meyer, Phys. Rev. C37 N5 (1988) 2170
- [9] K. Jammari, R. Piepenbring and B. Silvestre-Brac, Phys. Rev. C 28, N5 (1983).
- [10] R. Julin, et al. Physics Letters 94B N2 (1980) 123
- [11] A. Passoja, R. Julin, J. Kantele and M. Luontama and M. Vergnes, Nucl. Phys. A441 (1985) 261.
- [12] J. Kantele et al. Z. Physik A289 (1979) 157 R. Julin, et al. Z. Physik A295 (1980) 315 R. Julin, et al. Z. Physik A303 (1981) 147
- [13] K. Heyde, R.A. Meyer, Phys. Rev. C42 N2 (1990) 790
- [14] E. A. McCutchan and N. V. Zamfir, Phys. Rev. C71 054306 (2005)
- [15] A. Passoja, J. Kantele, M. Luontama, J. Kumpulainen, R. Julin, P. O. Lipas and P. Toivonen, Physics Letters 124B, N3,4 (1983).
- [16] M. Luontama et al. Z. Physik A324 (1986) 317
- [17] E. L. Church and J. Weneser, Phys. Rev. 103 N4 (1956); A. Passoja et al. J. Phys. G 12 (1986) 1047
- [18] Vladimir P. Garistov IJMP E V.4 N2. (1995), 371; A.A. Solnyshkin, et al. Phys. Rev. C. 2005. V.72. P. 064321-1.
- [19] V. P. Garistov, P. Terziev, nucl-th/9811100 - Remark to "On the Description of Fermion Systems in Boson Representations".
- [20] Abdurazakov A.A. et al. Beta-spectrographs with constant magnets, Tashkent, Uzbekistan, 1972.
- [21] Egorov O.K. et al. JTF. 2003. V. 48, ? 3. 4. I. Adam et al. Izv. RAN, ser. Fiz. 2002. V.66. P.1384 and C.W. Reich, Nuclear Data Sheets 105, 557 (2005).
- [22] I. Adam et al. Izv. RAN, ser. Fiz. 2002. V.66. P.1384 and C.W. Reich, Nuclear Data Sheets 105, 557 (2005).















